



## Positive streamer initiation from raindrops in thundercloud fields

Babich, L. P.; Bochkov, E. I.; Kutsyk, I. M.; Neubert, Torsten; Chanrion, Olivier

*Published in:*  
Journal of Geophysical Research: Atmospheres

*Link to article, DOI:*  
[10.1002/2016JD024901](https://doi.org/10.1002/2016JD024901)

*Publication date:*  
2016

*Document Version*  
Publisher's PDF, also known as Version of record

[Link back to DTU Orbit](#)

*Citation (APA):*  
Babich, L. P., Bochkov, E. I., Kutsyk, I. M., Neubert, T., & Chanrion, O. (2016). Positive streamer initiation from raindrops in thundercloud fields. *Journal of Geophysical Research: Atmospheres*, 121(11), 6393-6403. <https://doi.org/10.1002/2016JD024901>

---

### General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

## RESEARCH ARTICLE

10.1002/2016JD024901

## Key Points:

- Positive streamers can be initiated from charged raindrops in sub-breakdown electric field
- The charge and drop geometry play an important role in streamer development
- The field required for streamer formation is higher than the measured thundercloud fields

## Correspondence to:

L. P. Babich,  
babich@elph.vniief.ru

## Citation:

Babich, L. P., E. I. Bochkov, I. M. Kutsyk, T. Neubert, and O. Chanrion (2016), Positive streamer initiation from raindrops in thundercloud fields, *J. Geophys. Res. Atmos.*, 121, 6393–6403, doi:10.1002/2016JD024901.

Received 5 FEB 2016

Accepted 17 MAY 2016

Accepted article online 19 MAY 2016

Published online 9 JUN 2016

## Positive streamer initiation from raindrops in thundercloud fields

L. P. Babich<sup>1</sup>, E. I. Bochkov<sup>1</sup>, I. M. Kutsyk<sup>1</sup>, T. Neubert<sup>2</sup>, and O. Chanrion<sup>2</sup>
<sup>1</sup>Russian Federal Nuclear Center-VNIIEF, Nizhni Novgorod Oblast, Russia, <sup>2</sup>National Space Institute, Technical University of Denmark, Kgs. Lyngby, Denmark

**Abstract** The threshold field for the electric gas discharge in air is  $\approx 26 \text{ kVcm}^{-1} \text{atm}^{-1}$ , yet the maximum field measured (from balloons) is  $\approx 3 \text{ kVcm}^{-1} \text{atm}^{-1}$ . The question of how lightning is stimulated is therefore one of the outstanding problems in atmospheric electricity. According to the popular idea first suggested by Loeb and developed further by Phelps, lightning can be initiated from streamers developed in the enhanced electric field around hydrometeors. In our paper, we prove by numerical simulations that positive streamers are initiated, specifically, around charged water drops. The simulation model includes the kinetics of free electrons, and positive and negative ions, the electron impact ionization and photon ionization of the neutral atmospheric constituents, and the formation of space charge electric fields. Simulations were conducted at air pressure 0.4 atm, typical at thundercloud altitudes, and at different background electric fields, drop sizes, and charges. We show that the avalanche-to-streamer transition is possible near drops carrying 63–485 pC in thundercloud fields with intensity of  $10 \text{ kVcm}^{-1} \text{atm}^{-1}$  and  $15 \text{ kVcm}^{-1} \text{atm}^{-1}$  for drops sizes of 1 mm and 0.5 mm, respectively. Thus, the electric field required for the streamer formation is larger than the measured thunderstorm fields. Therefore, the results of simulations suggest that second mechanisms must operate to amplify the local field. Such mechanisms could be electric field space variations via collective effects of many hydrometeors or runaway breakdown.

## 1. Introduction

Lightning discharge initiation remains one of the unresolved problems in the physics of atmospheric electricity despite a long history of research. A two-stage initiation process is commonly accepted [e.g., Petersen *et al.*, 2008]. In the first stage, the thundercloud electric field is amplified locally to the threshold field for the breakdown,  $E_{br}$ , where the source rate of electrons from ionization of neutral molecules equals the loss rate of electrons from their attachment to oxygen molecules. For fields above the threshold, electron avalanches form and transition into streamers can occur. The threshold field is proportional to the air density. For dry air at standard temperature and pressure (STP) conditions the threshold is  $E_{br} \approx 26 \text{ kVcm}^{-1}$  [Raizer, 1991, p. 427], scaling to  $\approx 10 \text{ kVcm}^{-1}$  at 0.4 atm typical at thundercloud altitudes. The reduced field, independent of the density (pressure), is in both cases  $\approx 26 \text{ kVcm}^{-1} \text{atm}^{-1}$ . During the second stage, a multitude of streamers forming the streamer corona heats the atmospheric gas and a hot leader channel is developed. Most research in lightning initiation is devoted to the first stage [Loeb, 1966; Phelps, 1974; Griffiths and Phelps, 1976; Nguyen and Michnowski, 1996; Blyth *et al.*, 1998; Gurevich *et al.*, 1997, 1999; Solomon *et al.*, 2001; Dwyer, 2005; Petersen *et al.*, 2008; Babich *et al.*, 2009, 2011, 2012; Dubinova *et al.*, 2015; Sadighi *et al.*, 2015].

The main difficulty with the first stage is that the electric fields measured in thunderclouds do not exceed  $3\text{--}4 \text{ kVcm}^{-1} \text{atm}^{-1}$  [Marshall *et al.*, 1995, 2005; Thomas and Rison, 2005], which is significantly below the threshold. Nevertheless, to initiate lightning, the field must be above the threshold, at least locally. One problem in measuring atmospheric DC electric fields is the electromagnetic noise connected with fluctuations in the space charge density caused by inhomogeneities of the air, ionization sources, etc. To counter this, probes have spheres with radii that are large relative to the spatial scales of the worst fluctuations. For balloon measurements, boom lengths are typically larger than 1 m and sphere diameters are larger than 10 cm. Therefore, such sensors cannot resolve scales below  $\sim 1$  m. In addition, the temporal resolution of balloon experiments  $\sim 0.1$  s [e.g., Marshall *et al.*, 2005] is far from the requirements for detection of the transient fields of the streamer initiation.

The most accepted and fundamental building block of lightning initiation is the field enhancement from hydrometeor seeds where the thunderstorm field is enhanced in the vicinity of hydrometeors as a result of

their polarization or accumulated electric charge [Blyth *et al.*, 1998]. This mechanism simulated and discussed in our paper is at the smallest scales of millimeters and nanoseconds.

Another proposed process for thundercloud field intensification is connected with the relativistic runaway electron avalanche (RREA) seeded by MeV-energy electrons of cosmic ray air showers [Gurevich *et al.*, 1997]. However, numerical simulations [Babich *et al.*, 2009, 2012] demonstrate that the achievable field is  $\approx 8.5 \text{ kVcm}^{-1} \text{ atm}^{-1}$  and that the external field must be  $\approx 4 \text{ kVcm}^{-1} \text{ atm}^{-1}$  over a length of 2 km, which surpasses a fundamental limit of the size and extension of the electric field in air [Dwyer, 2003].

The stationary background cosmic radiation in a thundercloud electric field can also be a source of RREAs. An RREA may lead to the formation of a conducting channel sprouting downward from the upper edge of the negative cloud region to the bottom edge of the positive region. During the channel development, a field at its front is amplified and for a time  $\sim 10 \text{ s}$  the electric field magnitude reaches values close to the breakdown threshold,  $E_{br}$ , in a region of dimension  $\sim 10 \text{ m}$  [Babich *et al.*, 2011, 2012]. However, this requires rather large rates of cloud electrification,  $k_{ch} = 2\text{--}13 \text{ Cs}^{-1}$ . With more realistic rates,  $k_{ch} = 0.3\text{--}0.6 \text{ Cs}^{-1}$ , the maximum value of a field is  $6\text{--}16 \text{ kVcm}^{-1} \text{ atm}^{-1}$  [Dwyer, 2005; Babich *et al.*, 2011, 2012].

Our proposition is that streamers in thunderstorms originate near hydrometeors in local electric field regions where an ambient electric field is amplified either by the RREA mechanism seeded by the background cosmic ray flux [Dwyer, 2005; Babich *et al.*, 2011, 2012] or by cloud charge fluctuations arising from a local assemblage of charged hydrometeors [Trautenberg and Iudin, 2005].

We focus on the main process of streamers initiation near charged hydrometeors in thunderstorm electric fields by discussing two-dimensional simulations of streamer initiation from charged water drops for a range of parameters. Our work complements the studies by Sadighi [2015] and Dubinova *et al.* [2015]. Section 2 describes the mathematical formulation, section 3 the conditions for streamer initiation, and section 4 the simulation approach. Results are presented in section 5, and discussions and conclusions are in section 6.

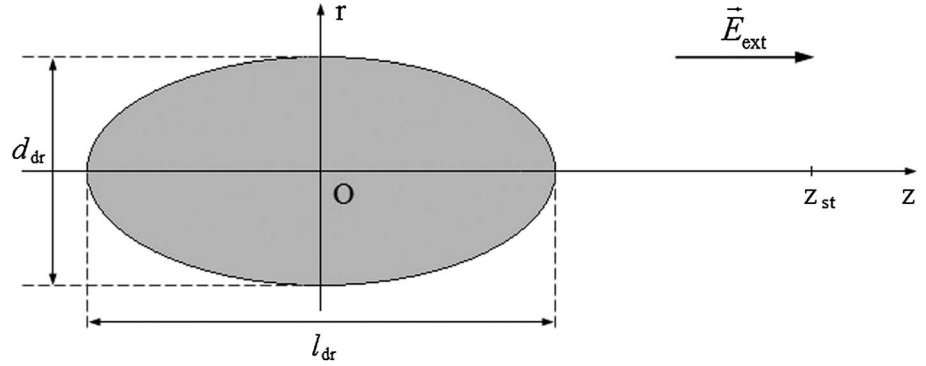
## 2. The Mathematical Formulation

The kinetics of electrons (e), positive (p), and negative (n) ions in a streamer discharge is described by the conventional set of equations:

$$\begin{aligned} \frac{\partial n_e}{\partial t} + \text{div } \vec{j}_e &= (\alpha_{ion} - \alpha_{att}) |\vec{j}_e| - \beta_{ep} n_e n_p + S_{ph}, \\ \frac{\partial n_p}{\partial t} + \text{div } \vec{j}_p &= \alpha_{ion} |\vec{j}_e| - \beta_{ep} n_e n_p - \beta_{pn} n_p n_n + S_{ph}, \\ \frac{\partial n_n}{\partial t} + \text{div } \vec{j}_n &= \alpha_{att} |\vec{j}_e| - \beta_{pn} n_p n_n, \end{aligned} \quad (1)$$

where  $n_e$ ,  $n_p$ ,  $n_n$  are the number densities;  $\vec{j}_e = -n_e \mu_e \vec{E} - D_e \nabla n_e$ ,  $\vec{j}_p = n_p \mu_p \vec{E}$ , and  $\vec{j}_n = -n_n \mu_n \vec{E}$  are the flux densities of the particles;  $\mu_e$  and  $\mu_{p,n}$  are the electron and ion mobilities;  $D_e$  is the electron diffusion coefficient;  $\alpha_{ion}$  is the impact ionization coefficient, and  $\alpha_{att}$  is the coefficient of electron attachment to oxygen molecules;  $\beta_{ep}$  and  $\beta_{pn}$  are the coefficients of recombination of electrons and positive ions and positive and negative ions and  $S_{ph}$  is the photoionization source. Although photoionization is small in comparison with the ionization produced by electron impacts, it is necessary for the positive streamer propagation because the streamer front moves in the opposite direction of the electrons. Photoionization initiates electron avalanches ahead of the ionization wave, thereby facilitating its propagation. The ionization coefficient, the electron mobility, and diffusion coefficients as functions of the reduced electric field  $E/P$  for the air (78.11%  $N_2$ , 20.91%  $O_2$ , 0.98%  $Ar$ ) were calculated using BOLSIG+ program [Hagelaar and Pitchford, 2005]. The other coefficients used are the same as in papers [Babich *et al.*, 2014, 2015]. The photoionization source  $S_{ph}$  is computed using a model by Bourdon *et al.* [2007], in which a calculation of the integral in the classic model by Zheleznyak *et al.* [1982] is replaced with a solution of three Helmholtz's equations.

We investigate a formation of positive streamers around water drops carrying a positive charge  $Q_{dr}$  in an external electric field. While falling in a vertical electric field the drops are extended along the vertical direction [Shishkin, 1964, page 326]. Therefore, we model drops as rotation ellipsoids with a minor semiaxis,



**Figure 1.** The raindrop model.

$d_{dr}/2$  and major semiaxis,  $l_{dr}/2$ , aligned with the vector of an external uniform electric field  $\vec{E}_{ext}(z, r) = E_{ext}\vec{e}_z$  (Figure 1). The vector  $\vec{e}_z$  is directed downward as in typical thunderclouds where the positive charge is located in the cloud top and the negative charge in the bottom; hence, the axis OZ in Figure 1 also is directed downward. In view of the cylindrical symmetry, the problem is two-dimensional and is solved in cylindrical coordinates  $(z, r)$ .

In an external field, a drop is polarized. As a result, the positive charge is concentrated at the ellipsoid bottom and the negative at the top. The charge motion in the drop is described by the following set of equations:

$$\begin{aligned} \frac{\partial \rho_{dr}}{\partial t} + \text{div } \vec{J}_{dr} &= 0, \\ \vec{J}_{dr} &= \sigma_{dr} \vec{E}, \end{aligned} \quad (2)$$

where  $\vec{J}_{dr}$ ,  $\rho_{dr}$ ,  $\sigma_{dr}$  are the current density, charge density, and electrical conductivity in the drop. The raindrop conductivity is within the limits from  $5 \cdot 10^{-4}$  to  $100 \cdot 10^{-4} \text{ Sm}^{-1}$  [Muchnik, 1974, p. 167] and we use

$$\sigma_{dr}(z, r) = \begin{cases} 100 \cdot 10^{-4} \text{ S} \cdot \text{m}^{-1}, & (z, r) \in D_{dr}, \\ 0, & (z, r) \notin D_{dr}. \end{cases} \quad (3)$$

Here  $D_{dr}$  is the space domain occupied by the ellipsoid (drop). The background conductivity inside thunderstorms ( $\sim 10^{-13} \text{ Sm}^{-1}$ ) [Rycroft et al., 2007] at an altitude of 7 km can be neglected.

The set of equations (1) and (2) is closed by the equations for the self-consistent electric field

$$\begin{aligned} \text{div}(\varepsilon \vec{\nabla} \phi) &= \frac{\rho_{dr}}{\varepsilon_0} - \frac{\rho_{dis}}{\varepsilon_0}, \\ \vec{E} &= -\vec{\nabla} \phi, \end{aligned} \quad (4)$$

where  $\rho_{dis} = e(n_p - n_n - n_e)$  is the charge density in the avalanche/streamer,  $e$  is the elementary charge,  $\varepsilon_0$  is the dielectric permittivity of the free space, and  $\varepsilon$  is the media relative permittivity

$$\varepsilon(z, r) = \begin{cases} 1, & (z, r) \notin D_{dr}; \\ 80, & (z, r) \in D_{dr}, \end{cases} \quad (5)$$

where 1 and 80 are magnitudes of the dielectric permittivity of air and water correspondingly.

Let us now formulate the boundary conditions for equations (1), (2) and (4). On the border of the computation domain,  $G_{sim}$ , zero boundary conditions for equations (1) are accepted

$$\vec{J}_e(z, r) = \vec{J}_p(z, r) = \vec{J}_n(z, r) = \vec{0}, \quad (z, r) \in G_{sim}. \quad (6)$$

On the drop surface,  $G_{dr}$ , we accept the boundary condition according to which charged particles (electrons and ions) penetrate into the drop volume if the particle velocity vector is directed inward the drop, otherwise  $\vec{j} = 0$ ; i.e., particle do not leave the drop.

The boundary condition for equation (2) on  $G_{dr}$  is as follows:

$$\vec{J}_{dr}(z, r) = \begin{cases} e \cdot \vec{J}_p(z, r), & (\vec{E}(z, r) \cdot \vec{n}_{dr}(z, r)) < 0; \\ -e \cdot (\vec{J}_e(z, r) + \vec{J}_n(z, r)), & (\vec{E}(z, r) \cdot \vec{n}_{dr}(z, r)) > 0. \end{cases} \quad (z, r) \in G_{dr}; \quad (7)$$

Here  $\vec{n}_{dr}$  is a normal to the drop outer surface.

The electric potential  $\phi$  on the border of the computation domain  $G_{sim}$  is a sum of the external field potential  $\phi_{ext}$  and the potential of the space charge field  $\phi_{int}$ :

$$\phi(z, r) = \phi_{ext}(z, r) + \phi_{int}(z, r), \quad (z, r) \in G_{sim}. \quad (8)$$

The  $\phi_{ext}$  satisfies the equation  $\vec{E}_{ext} = -\vec{\nabla} \phi_{ext}$  and  $\phi_{int}$  is given by the Poisson's equation:

$$\phi_{int}(z, r) = \frac{1}{4\pi\epsilon_0} \int_{D_{sim}} \frac{(\rho_{dr}(z', r') + \rho_{dis}(z', r')) \cdot 2\pi r' dr' dz'}{\sqrt{(z - z')^2 + (r - r')^2}}, \quad (9)$$

where  $D_{sim}$  a volume of the computation domain.

### 3. The Condition for the Streamer Initiation

At least one free electron is required to start an electron avalanche (in the atmosphere free electrons are permanently produced by cosmic rays and natural radioactivity). If the external field intensity exceeds  $E_{br}$ , the primary electron initiates an electron avalanche. When the avalanche has grown to a stage where the space charge field of the avalanche is comparable to the background field, the avalanche transition into a streamer (an ionization wave) can occur. The well-known criterion for the transition in a gas discharge gap is [Raizer, 1991, p. 426]

$$(\alpha_{ion}(E_{ext}) - \alpha_{att}(E_{ext}))d \approx 18 - 20, \quad (10)$$

where  $d$  is the interelectrode spacing and  $E_{ext}$  is the electric field intensity in the gap.

In our case, the electric field is strongly inhomogeneous. Its intensity maximizes at the ellipsoid tops and rapidly decreases with distance from the ellipsoid, reaching  $E_{ext}$  at infinity. Accordingly, we reformulate condition (10) as follows:

$$\int_{l_{dr}/2}^{z_{in}} (\alpha_{ion}(E(z, r=0), P_g) - \alpha_{att}(E(z, r=0), P_g)) dz = 20, \quad (11)$$

where  $z_{in}$  is the coordinate at which  $\alpha_{ion} = \alpha_{att}$  and  $P_g$  is the gas pressure.

The minimum value of the field intensity at which a cathode-directed (positive) streamer can develop in air at STP is  $4.65 \text{ kVcm}^{-1}$  [Bazelyan and Raizer, 2001, p. 55]. We therefore chose to simulate the reduced external electric field intensities  $E_{ext}/P_g = 5, 8, 10, 15$  and  $20 \text{ kVcm}^{-1} \text{ atm}^{-1}$ . With the used in our simulations, air pressure  $P_g = 0.4 \text{ atm}$ , corresponding to the altitude of  $\approx 8 \text{ km}$  above sea level, magnitudes of the field strength  $E_{ext}$  are equal to 2, 3.2, 4, 6 and  $8 \text{ kVcm}^{-1}$  accordingly.

At fixed drop dimensions ( $d_{dr}, l_{dr}$ ) and in a given  $E_{ext}$ , the drop charge,  $Q_{dr}$ , is the unique parameter defining the field intensity distribution. Hence, there exists a charge  $Q_{dr}$  for which condition (11) will be satisfied. To find the required  $Q_{dr}$ , we, in the absence of the background plasma ( $\rho_{dis} = 0$ ), calculated the stationary drop charge distribution in an external field. With this goal, we solve equations (2) and (4). As the stationary charge distribution in the given external field is unique, it does not depend on the choice of the initial charge distribution. We use the following initial conditions with homogeneous charge density in the drop:

$$\begin{aligned} \rho_{dr}(z, r, 0) &= Q_{dr}/V_{dr}, \\ \vec{E}(z, r, 0) &= \vec{E}_{ext}, \end{aligned} \quad (12)$$

where  $V_{dr}$  is the drop volume. In reality, the charge always is distributed along the drop surface. Therefore, the first equation in equation (12) is not physical; we use it for the sake of simplicity just to start simulations.

**Table 1.** Drop Charge  $Q_{dr}$  With Which the Condition (11) is Satisfied. Parameters Characterizing Drop Stability Conditions (13)–(15)  $W$ ,  $w$ ,  $X$ . Streamer Velocity  $v_f$ .  $P_g = 0.4$  atm

$d_{dr}$ (mm)	$l_{dr}$ (mm)	$E_{ext}$ (kVcm <sup>-1</sup> )	$Q_{dr}$ (pC)	$W$	$w$	$X$	$v_f^a$ (ms <sup>-1</sup> )
1.5	1.5	2	400	0.88	0.04	<b>1.01</b>	$<1.2 \cdot 10^4$
		3.2	377	0.78	0.11	0.95	$<2.9 \cdot 10^4$
		4	355	0.69	0.17	0.87	$1.4 \cdot 10^5$
		6	315	0.55	0.39	0.79	$2.8 \cdot 10^5$
		8	270	0.40	0.69	0.73	$3.9 \cdot 10^5$
	3.0	2	485	0.32	0.07	0.34	$<1.3 \cdot 10^4$
		3.2	445	0.27	0.18	0.32	$6.9 \cdot 10^4$
		4	405	0.22	0.28	0.29	$1.5 \cdot 10^5$
		6	340	0.16	0.62	0.31	$2.8 \cdot 10^5$
		8	250	0.09	1.10	0.37	$3.8 \cdot 10^5$
	1.0	2	220	0.90	0.03	<b>1.00</b>	$<6.9 \cdot 10^3$
		3.2	210	0.82	0.07	0.95	$<9.8 \cdot 10^3$
		4	200	0.74	0.12	0.88	$1.1 \cdot 10^5$
		6	178	0.59	0.26	0.77	$2.2 \cdot 10^5$
		8	157	0.46	0.46	0.69	$3.1 \cdot 10^5$
		2	270	0.34	0.05	0.35	$<7.7 \cdot 10^3$
		3.2	252	0.29	0.12	0.32	$<1.3 \cdot 10^4$
		4	235	0.26	0.18	0.30	$8.6 \cdot 10^4$
		6	200	0.18	0.41	0.28	$2.1 \cdot 10^5$
		8	165	0.13	0.73	0.30	$3.0 \cdot 10^5$
		2	80	0.95	0.01	<b>1.06</b>	$<1.4 \cdot 10^4$
		3.2	77	0.88	0.04	0.99	$<4.0 \cdot 10^4$
0.5	0.5	4	75	0.84	0.06	0.95	$6.8 \cdot 10^4$
		6	69	0.71	0.13	0.84	$1.3 \cdot 10^5$
		8	63	0.59	0.23	0.75	$2.0 \cdot 10^5$
		2	103	0.39	0.02	0.40	$<3.0 \cdot 10^3$
		3.2	98	0.36	0.06	0.37	$<6.4 \cdot 10^4$
	1.0	4	93	0.32	0.09	0.35	$8.4 \cdot 10^4$
		6	84	0.26	0.21	0.31	$1.4 \cdot 10^5$
		8	74	0.20	0.37	0.29	$2.1 \cdot 10^5$

<sup>a</sup>The sign "<" means that the velocity does not reach a stationary value.

The time step used here to solve equation (2) was set  $\Delta t = 0.1 \tau_m$ , where  $\tau_m = (\epsilon_{\text{water}} \epsilon_0) / \sigma_{dr} \approx 70$  ns is the Maxwellian electric field relaxation time inside the drop. The simulations were run until the field distribution was stationary (simulation time  $t_{\text{run}} \approx 10 \tau_m$ ). The various configurations for which condition (11) is satisfied are shown in Table 1. The values of  $Q_{dr}$  are quite realistic. They are in the range of measured charge on particles with 1–3 mm diameter inside thunderclouds, which are between 10 and 200 pC with a few particles having charges from 200 to 400 pC [Marshall and Winn, 1982]. In Figure 2 a relaxation process of the space charge density and electric field inside the drop is illustrated for the case  $d_{dr} = 1.0$  mm,  $l_{dr} = 1.0$  mm,  $Q_{dr} = 200$  pC,  $E_{\text{ext}} = 4$  kVcm<sup>-1</sup>. It is seen, at the end of the simulation the drop charge accumulates in thin presurface layer and, as a result, the electric field in the drop reduces almost to zero with the exception of the presurface layer.

The obtained stationary drop charge density and electric field distribution are used as initial in the next section while simulating the streamer discharge development.

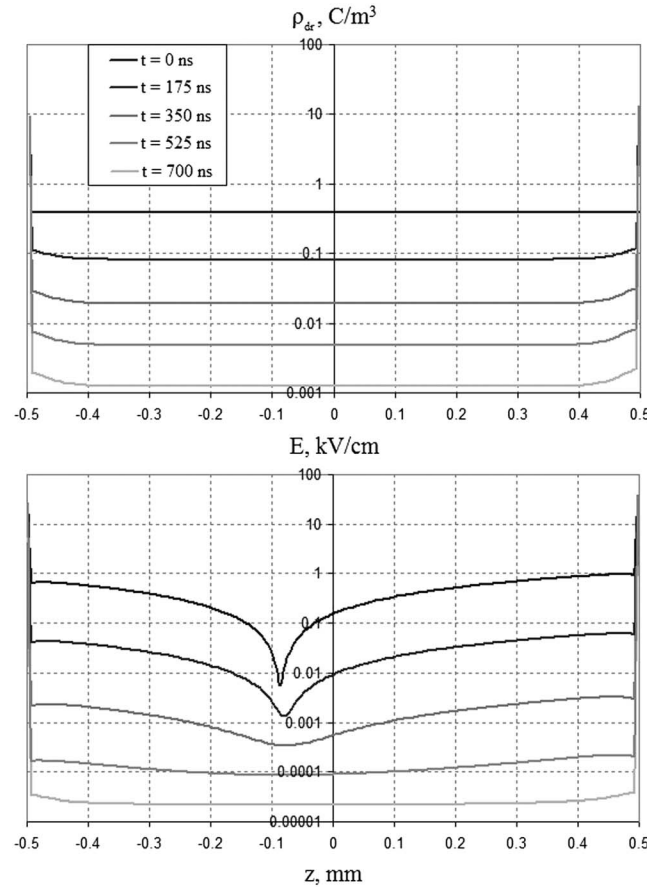
As charged drops in an external electric field may be unstable, it is necessary to analyze this problem. A charged ball drop in the absence of an external field is unstable if the Rayleigh's criterion is satisfied [Rayleigh, 1882]:

$$W = \frac{Q_{dr}}{64\pi^2 \epsilon_0 \sigma r_{dr}^3} \geq 1, \quad (13)$$

where  $Q_{dr}$  and  $r_{dr}$  are the drop charge and radius and  $\sigma$  is the surface tension.

An uncharged drop in a uniform electric field is unstable if the Taylor's criterion is satisfied [Taylor, 1964]:

$$w = \frac{4\pi \epsilon_0 E^2 r_{dr}}{\sigma} \geq 2.6 \quad (14)$$



**Figure 2.** Evolution of (top) the space charge density and (bottom) the field intensity on the symmetry axis inside the drop.  $d_{dr} = l_{dr} = 1.0$  mm,  $Q_{dr} = 200$  pC,  $E_{ext} = 4$  kVcm<sup>-1</sup>.

1.0, and 1.5 mm, respectively, and  $\Delta t = 2 \cdot 10^{-13}$  s. The numerical diffusion when calculating the flux terms in equation (1) is minimized using the method by Zalesak [1979] in the same way as in [Babich et al., 2014]. The stationary electric field around the charged ellipsoid and drop charge density at  $t = 0$  are determined as described in the previous section.

Although an electron avalanche can be initiated by one electron in the drop vicinity, however, fluid equations describe the evolution of a large number of particles. The conventional solution to this dilemma is to prescribe the initial conditions  $n_e(z, r, 0) = n_p(z, r, 0) = \delta(z - z_{in})$ , but this leads to too large electron diffusion at the initial stage of the avalanche. Instead, we set  $n_e(z, r, 0) = n_p(z, r, 0) = N_e^0 \delta(z - z_{st})$ , where  $N_e^0$  is set 100 and  $z_{st}$  is found from the equation:

$$\int_{l_{dr}/2}^{z_{st}} (\alpha_{ion}(E(z, r = 0), P_g) - \alpha_{att}(E(z, r = 0), P_g)) dz = 20 - \ln(N_e^0). \quad (16)$$

Condition (16) allows preserving condition (11), and our approach is simply not to follow the early stage of the avalanche. As a result, the initial conditions for (equation (1)) are as follows:

$$\begin{aligned} n_e(z, r, 0) &= N_e^0 \delta(z - z_{st}), \\ n_p(z, r, 0) &= n_e(z, r, 0), \\ n_n(z, r, 0) &= 0. \end{aligned} \quad (17)$$

Simulations executed with  $N_e^0 = 1000$  did not alter significantly results. Discharge simulations are run until  $t_{run} = 40$  ns or until the streamer front reaches the simulation domain boundary.

For a charged drop in a uniform field, the instability criterion by Grigor'ev and Shiryayeva [1989] holds

$$X \equiv (1 + 2.07e_0^2)W + 0.092(1 + 6.62e_0^2)w \geq 1, \quad (15)$$

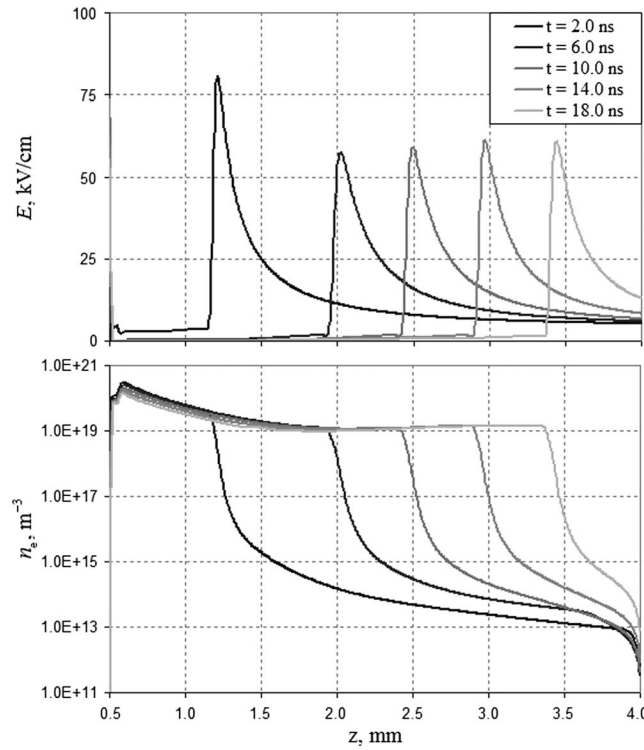
where  $e_0^2 = 0.18w/(1 - W)$ .

In Table 1, the magnitudes of  $W$ ,  $w$ , and  $X$  are presented for an accepted magnitude of the surface tension  $\sigma = 77$  Nm<sup>-1</sup> for supercooled water at  $-10^\circ\text{C}$  [Hruby et al., 2014]. It is seen that the drops are stable relative to the criteria (13) and (14) but marginally unstable according to criterion (15) in some configurations if the field is 2 kVcm<sup>-1</sup>. However, as shown later (see the last paragraph of section 5), streamer initiation in these configurations is unlikely; therefore, in these cases a question about drops stability is irrelevant.

#### 4. The Simulation Approach to the Streamer Discharge Development

To simulate the streamer discharge, equations (1), (2), and (4) are solved using a homogeneous square spatial mesh with steps  $\Delta z = \Delta r = 2.5 \cdot 10^{-6}$ ,  $5 \cdot 10^{-6}$ , and  $5 \cdot 10^{-6}$  m for  $d_{dr} = 0.5$ ,





**Figure 3.** Evolution of (top) the field intensity and (bottom) the electron number density on the discharge symmetry axis. Parameters  $d_{dr} = l_{dr} = 1.0$  mm,  $E_{ext} = 4$  kVcm $^{-1}$ ,  $P_g = 0.4$  atm.

## 5. Results

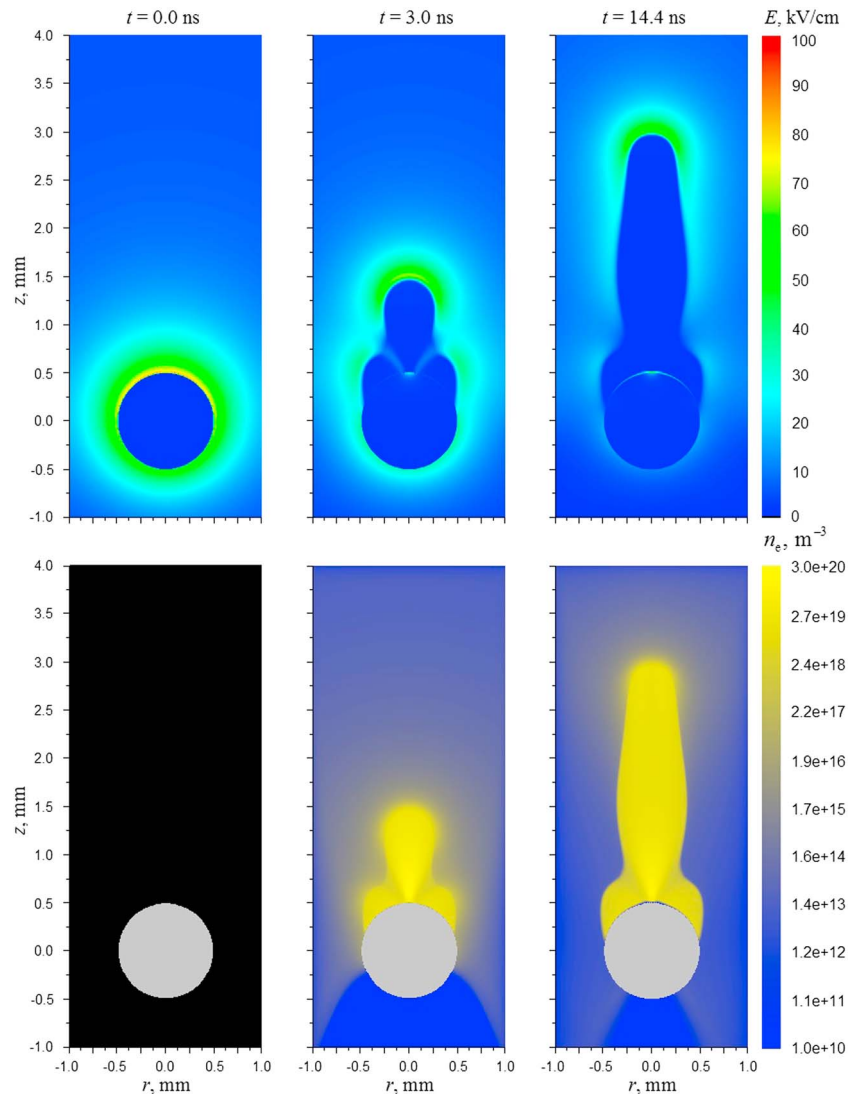
In the following, we define the streamer front position as the location of the maximum electric field. Initially, the electron avalanches develop from  $z = z_{st}$  toward the drop, whereas photoionization creates new free electrons also at farther distance from the drop, which in turn may develop avalanches toward the drop. In all the simulated configurations, the streamer front passed  $z = z_{in}$ , i.e., entered into the domain where at the initial moment of time  $\alpha_{ion} < \alpha_{att}$ . The distributions of the field intensity and electron number density along the discharge symmetry axis are shown as functions of time in Figure 3. They have the typical characteristics of a streamer. Scaled to STP conditions the electron number density in the channel is  $\sim 10^{20}$  m $^{-3}$  (the electron number density scales as  $\sim P_g^2$  [Pasko et al., 1998]) and the reduced field intensity at the streamer front is  $\approx 160$  kVcm $^{-1}$ atm $^{-1}$ . When the streamer propagates

farther from the drop, the front field decreases. The streamer leaving the enhanced field region near the drop and an increase in the streamer radius cause this. The electric field and electron densities in the  $(z, r)$  plane are illustrated in Figure 4 for  $d_{dr} = 1.0$  mm,  $l_{dr} = 1.0$  mm,  $E_{ext} = 4$  kVcm $^{-1}$  at three different times. It is seen that the radius of the streamer channel is growing, reaching  $\approx 0.25$  mm. The plots of Figures 3 and 4 are typical for all configurations in Table 1.

As the streamer propagates, negative charge is transferred from the streamer channel to the drop, which reduces the initially positive charge on the drop and the field around the drop. The question is now if the streamer will continue to propagate after the positive drop charge is annihilated or after the streamer front leaves the domain of enhanced field around the drop. To address this question, we show in Figure 5 the time dependences of the streamer velocity, the front field intensity, the number of free electrons, and the drop charge for  $d_{dr} = 1.0$  mm,  $l_{dr} = 1.0$  mm,  $E_{ext} = 4$  kVcm $^{-1}$ . We note that the positive drop charge disappears at  $t \approx 15$  ns and afterward the drop very slowly accumulates negative charge. The field intensity at the streamer front achieves a stationary magnitude  $E_f \approx 62$  kVcm $^{-1}$  at  $t \approx 12$  ns and the electron number approaches a linear growth rate after  $t \approx 8$  ns. The streamer velocity initially decreases and approaches a stationary magnitude equal  $v_f \approx 1.1 \cdot 10^5$  ms $^{-1}$  toward the end of the run, which is consistent with the observed minimum value of the streamer speed at STP conditions  $v_{s,min} \approx 10^5$  ms $^{-1}$  [Briels et al., 2008]. Noting that our simulations were carried out for  $P_g = 0.4$  atm and that there is some uncertainty of the  $v_{s,min}$  magnitude, we conclude that for the configuration shown in Figures 3 and 4, the streamer will continue to propagate.

The final streamer velocity for all the simulated configurations is given in Table 1. It is seen that for all configurations with  $E_{ext} = 2$  kVcm $^{-1}$  and  $3.2$  kVcm $^{-1}$  the velocity is much less than  $v_{s,min}$ . In addition, we find that at some point in time, the number of electrons begins to decrease (not shown). This means that streamers will not be initiated at such fields. The conclusion follows from our model that stable propagation of streamers requires higher fields. Adopting the criterion that  $v_f > v_{s,min}$ , we find that streamer initiation is possible in external fields with strength  $E_{ext} \geq 4$  kVcm $^{-1}$  and  $6$  kVcm $^{-1}$  from drops with  $d_{dr} \geq 1.0$  mm,  $Q_{dr} \geq 200$  pC, and  $d_{dr} \geq 0.5$  mm,  $Q_{dr} \geq 70$  pC, respectively.



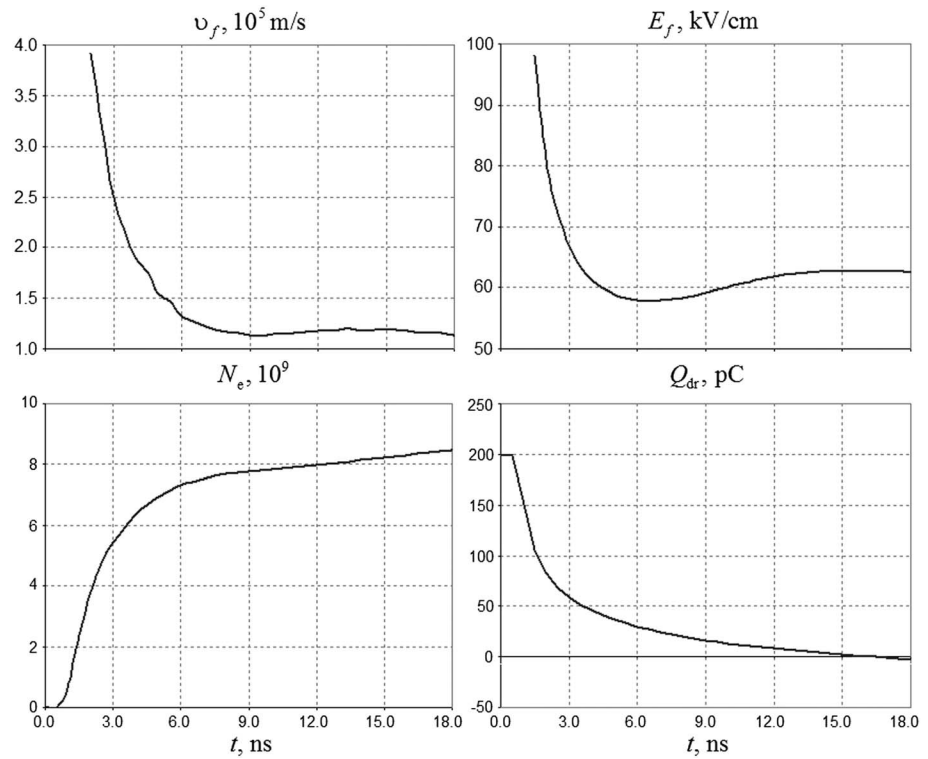


**Figure 4.** Two-dimensional distribution of (top) the field intensity and (bottom) the electron number density at various moments of time. Parameters  $d_{dr} = l_{dr} = 1.0$  mm,  $E_{ext} = 4$  kVcm $^{-1}$ ,  $P_g = 0.4$  atm.

## 6. Discussions and Conclusions

A number of numerical simulation studies have been carried out to study streamer discharge initiation in thundercloud electric fields. Solomon *et al.* [2001] presented results of one-dimensional simulations of positive discharge initiation at  $P_g = 0.5$  atm from hydrometeors. They found that streamers can be initiated in the following cases: (1) during collisions of water drops with radius of 2.7 mm and 0.65 mm in a field of 4 kVcm $^{-1}$ atm $^{-1}$ , (2) from isolated ice needles in a field of 22 kVcm $^{-1}$ atm $^{-1}$ , or (3) in a field of 18 kVcm $^{-1}$ atm $^{-1}$  if the needle is charged up to 100 pC.

In Sadighi [2015], a two-dimensional model of streamers initiated from uncharged dielectric hydrometeors in a uniform electric field is presented. Two cases were simulated: (1) a spherical ice hydrometeor with a radius 3.1 mm in an ambient electric field value of  $0.9 E_{br}$  and (2) a columniform liquid water filament with a length 2.9 mm and a radius 0.44 mm in a field of  $0.8 E_{br}$ . The initial seed electrons were a plasma cloud containing  $\sim 10^7$  electron-ion pairs. In both cases, a stable streamer discharge develops. Sadighi *et al.* [2015] study streamer formation from thundercloud hydrometeors in a field of  $0.3 E_{br}$ . The hydrometeor was represented by an ionized, cylindrical column with hemispherical caps containing an equal number of electrons and positive ions. The study concludes that the minimum length required for the streamer initiation is from 5 to 8 mm with a background



**Figure 5.** Time dependencies of the streamer velocity  $v_f$ , the field intensity at the streamer front  $E_f$ , the number of electrons in the discharge  $N_e$ , and the drop charge  $Q_{dr}$ . Parameters  $d_{dr} = l_{dr} = 1.0$  mm,  $E_{ext} = 4$  kVcm $^{-1}$ ,  $P_g = 0.4$  atm.

electron density of  $\sim 10^{15}$  m $^{-3}$ . In addition, it was found that the electron density must be nonuniform, otherwise the streamer branches. The authors suggested that such nonuniform spatial distribution could be created by previous corona discharges. However, replacing a hydrometeor with a strongly ionized patch of air the authors lost an opportunity of reproducing the most important process of the avalanche to streamer transition.

In *Dubnova et al.* [2015], results of simulations of positive streamer development are presented near an elongated ice hydrometeor, approximated as a prolate ellipsoid. Uncharged hydrometeor polarizes in the electric field due to the ice dielectric permittivity. To start the streamer process, a concentration of 100 cm $^{-3}$  of free seed electrons around the hydrometeor was assumed. This large concentration was hypothesized to be a result of a cosmic ray shower. The authors for a number of the field strength and hydrometeor length magnitudes computed a radius of the ellipsoid tip curvature required to fit the Meek number 10 (unlike the conventional magnitude 20: cf. equation (11)), which they state is sufficient for the streamer inception. Actually, *Dubnova et al.* simulated the streamer development near the hydrometeor of 6 cm length in the field of  $0.15 E_{br}$  and obtained a streamer moving with an average velocity of  $\approx 10^5$  ms $^{-1}$ .

As we can see, most published studies are focused on uncharged hydrometeors as a source of positive streamer origin. In contrast, our own work considers streamer initiation in self-consistent electric field in the vicinity of a *charged* raindrop. Unlike *Sadighi et al.* [2015], we started the simulation with a condition corresponding to only one seed electron in the hydrometeor vicinity and did not require enhanced (above the background) preionization. We found that avalanche-to-streamer transition is possible if the drop charge is in the range of  $Q_{dr} = 63\text{--}485$  pC. For drops with  $d_{dr} \geq 1.0$  mm and  $d_{dr} \geq 0.5$  mm streamers can be initiated and can propagate for  $E_{ext} \geq 4$  kVcm $^{-1}$  and 6 kVcm $^{-1}$ , respectively, corresponding to  $E_{ext} \approx 0.31 E_{br}$  and  $\approx 0.47 E_{br}$  (in our model  $E_{br} = 12.8$  kVcm $^{-1}$  at  $P_g = 0.4$  atm). The magnitudes of drop size and charge are quite realistic unlike in [*Dubnova et al.*, 2015], where extremely large hydrometeor dimensions were required to start the streamer discharge.

As in the earlier publications [*Sadighi*, 2015; *Sadighi et al.*, 2015; *Dubnova et al.*, 2015], we have simulated the important “smallest” seeding element. However, the problem remains that the scale is so small, and the

required fields are larger than observed fields, that a second stage of larger scale must be brought in play to amplify the field in larger volume. The scale of the second stage must also satisfy the limitation imposed by the spatial and temporal resolution of instruments measuring the intracloud electric field (see Introduction); otherwise, this stage would be detected. Collective effects of many hydrometeors and spatial variations in a cloud space charge density up to about 0.1–1 m (instrumental spatial resolution) could be causal to the development of enhanced fields in regions that larger than those of individual hydrometeors.

## Acknowledgments

The VNIIEF coauthors express the deepest gratitude to C. Haldoupis who, in cooperation with T. Neubert, has been an international collaborator in the completed ISTC project 3993–2010 and to N. Crosby, S. Cummer, A. van Deursen, J.R. Dwyer, R. Roussel-Dupre, D. Smith, T. Torii, and E. Williams for their support of the project proposal. They thank R.A. Roussel-Dupre and E.M.D. Symbalysty for the long-term collaboration, the continuation of which is this paper. The authors would like to thank the reviewers whose invaluable comments have allowed significant improvement of this paper. Readers can access the data from the paper via authors by emails: babich@elph.vniief.ru and neubert@space.dtu.dk. The work has benefited from collaborations within the European Science Foundation Research Networking Project “Thunderstorm Effects on the Atmosphere-Ionosphere System.”

## References

- Babich, L. P., E. I. Bochkov, and I. M. Kutsyk (2009), Numerical simulation of a conducting channel initiated by an extensive air shower in a thunderstorm electric field, *Geomagn. Aeron.*, 49(2), 232–238.
- Babich, L. P., E. I. Bochkov, and I. M. Kutsyk (2011), Lightning initiation mechanism based on the development of relativistic runaway electron avalanches triggered by background cosmic radiation. Numerical simulation, *J. Exp. Theor. Phys.*, 112(5), 902–909.
- Babich, L. P., E. I. Bochkov, J. R. Dwyer, and I. M. Kutsyk (2012), Numerical simulations of local enhancement of thundercloud field capable of lightning initiation trigger, allowing for the runaway electron avalanches seeded by cosmic rays, *J. Geophys. Res.*, 117, A09316, doi:10.1029/2012JA017799.
- Babich, L. P., E. I. Bochkov, and I. M. Kutsyk (2014), Mechanism of generation of runaway electrons in a lightning leader, *JETP Lett.*, 99(7), 386–390.
- Babich, L. P., E. I. Bochkov, I. M. Kutsyk, T. Neubert, and O. Chanrion (2015), A model for electric field enhancement in lightning leader tips to levels allowing X-ray and  $\gamma$ -ray emissions, *J. Geophys. Res. Space Physics*, 120, 5087–5100, doi:10.1002/2014JA020923.
- Bazelyan, E. M., and Y. P. Raizer (2001), *Fizika molnii i molniezaschita*, Fizmatlit, Moscow [in Russian]; (2000), in *Lightning Physics and Lightning Protection*, IOP, Bristol.
- Blyth, A. M., M. J. Christian, and J. Latham (1998), Corona emission thresholds for three types of hydrometeor interaction in thunderclouds, *J. Geophys. Res.*, 103(D12), 13,975–13,977, doi:10.1029/97JD02554.
- Bourdon, A., V. P. Pasko, N. Y. Liu, S. Celestin, P. Segur, and E. Marode (2007), Efficient models for photoionization produced by nonthermal gas discharges in air based on radiative transfer and the Helmholtz equations, *Plasma Sources Sci. Technol.*, 16, 656–678.
- Briels, T. M. P., S. Nijdam, E. M. van Veldhuizen, and U. Ebert (2008), Exploring streamer variability in experiments, *J. Phys. D Appl. Phys.*, 41, 234004.
- Dubnova, A., C. Rutjes, U. Ebert, S. Buitink, O. Scholten, and G. Thi Ngoc Trinh (2015), Prediction of lightning inception by large ice particles and extensive air showers, *Phys. Rev. Lett.*, 115, 015002, doi:10.1103/PhysRevLett.115.015002.
- Dwyer, J. R. (2003), A fundamental limit on electric fields in air, *Geophys. Res. Lett.*, 30(20), 2055, doi:10.1029/2003GL017781.
- Dwyer, J. R. (2005), The initiation of lightning by runaway air breakdown, *Geophys. Res. Lett.*, 32, L20808, doi:10.1029/2005GL023975.
- Griffiths, R., and C. Phelps (1976), A model for lightning initiation arising from positive corona streamer development, *J. Geophys. Res.*, 81, 3671–3676, doi:10.1029/JC081i021p03671.
- Grigor'ev, A. I., and S. O. Shiryayeva (1989), A mechanism of the step leader development and branching of intracloud linear lightning [In Russian], *Zh. Tekhnicheskoi Fiziki J. Tech. Phys.*, 59(5), 6–13.
- Gurevich, A. V., G. M. Milikh, and J. A. Valdivia (1997), Model of X-ray emission and fast preconditioning during a thunderstorm, *Phys. Lett. A*, 231, 402–408, doi:10.1016/S0375-9601(97)00354-X.
- Gurevich, A. V., K. P. Zybin, and R. A. Roussel-Dupre (1999), Lightning initiation by simultaneous effect of runaway breakdown and cosmic ray showers, *Phys. Lett. A*, 254, 79–87, doi:10.1016/S0375-9601(99)00091-2.
- Hagelaar, G. J. M., and L. C. Pitchford (2005), Solving the Boltzmann equation to obtain electron transport coefficients and rate coefficients for fluid models, *Plasma Sources Sci. Technol.*, 14, 722–733.
- Hruby, J., V. Vins, R. Mares, and J. Kalova (2014), Surface tension of supercooled water: No inflection point down to  $-25^{\circ}\text{C}$ , *J. Phys. Chem. Lett.*, 5(3), 425–428, doi:10.1021/jz402571a.
- Loeb, L. B. (1966), The mechanism of stepped and dart leaders in cloud-to-ground lightning strokes, *J. Geophys. Res.*, 71(20), 4711–4721, doi:10.1029/JZ071i020p04711.
- Marshall, T. C., and W. P. Winn (1982), Measurements of charged precipitation in a New Mexico thunderstorm: Lower positive charge centers, *J. Geophys. Res.*, 87, 7141–7157, doi:10.1029/JC087iC09p07141.
- Marshall, T. C., M. P. McCarthy, and W. D. Rust (1995), Electric field magnitudes and lightning initiation in thunderstorms, *J. Geophys. Res.*, 100(D4), 7097–7103, doi:10.1029/95JD00020.
- Marshall, T. C., M. Stolzenburg, C. R. Maggio, L. M. Coleman, P. R. Krehbiel, T. Hamlin, R. J. Thomas, and W. Rison (2005), Observed electric fields associated with lightning initiation, *Geophys. Res. Lett.*, 32, L03813, doi:10.1029/2004GL021802.
- Muchnik, B. M. (1974), *Physics of a Thunderstorm* [In Russian], Gydrometeoizdat ed., Leningrad.
- Nguyen, M. D., and S. Michnowski (1996), On the initiation of lightning discharge in a cloud 2. The lightning initiation on precipitation particles, *J. Geophys. Res.*, 101, 26,675–26,680, doi:10.1029/96JD02338.
- Pasko, V. P., U. S. Inan, and T. F. Bell (1998), Spatial structure of sprites, *Geophys. Res. Lett.*, 25, 2123–2126, doi:10.1029/98GL01242.
- Petersen, D., M. Bailey, W. H. Beasley, and J. Hallett (2008), A brief review of the problem of lightning initiation and a hypothesis of initial lightning leader formation, *J. Geophys. Res.*, 113, D17205, doi:10.1029/2007JD009036.
- Phelps, C. T. (1974), Positive streamer system intensification and its possible role in lightning initiation, *J. Atmos. Sol. Terr. Phys.*, 36, 103–111.
- Raizer, Y. P. (1991), *Gas Discharge Physics*, Springer, Berlin, doi:10.1007/978-3-642-61247-3.
- Rayleigh, L. (1882), On the equilibrium of liquid conducting masses charged with electricity, *Philos. Mag.*, 14(87), 184–186.
- Rycroft, M. J., A. Odzimek, N. F. Arnold, M. Füllekrug, A. Kulak, and T. Neubert (2007), New model simulations of the global atmospheric electric circuit driven by thunderstorms and electrified shower clouds: The roles of lightning and sprites, *J. Atmos. Sol. Terr. Phys.*, 69(17–18), 2485–2509, doi:10.1016/j.jastp.2007.09.004.
- Sadighi, S. (2015), Initiation of Streamers from Thundercloud Hydrometeors and Implications to Lightning Initiation. A dissertation submitted to the College of Science at Florida Institute of Technology in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Physics, Melbourne, Fla. [Available at <http://hdl.handle.net/11141/677>].
- Sadighi, S., N. Liu, J. R. Dwyer, and H. K. Rassoul (2015), Streamer formation and branching from model hydrometeors in subbreakdown conditions inside thunderclouds, *J. Geophys. Res. Atmos.*, 120, 3660–3678, doi:10.1002/2014JD022724.

- Shishkin, N. S. (1964), *Clouds, Precipitations and Thunderstorm Electricity* [In Russian], Gydrometeoizdat ed., Leningrad.
- Solomon, R., V. Schroeder, and M. B. Baker (2001), Lightning initiation: Conventional and runaway-breakdown hypothesis, *Q. J. R. Meteorol. Soc.*, *127*, 2683–2704.
- Taylor, G. (1964), Disintegration of water drops in an electric field, *Proc. R. Soc. A.*, *280*(1382), 383–397.
- Thomas, R. J., and W. Rison (2005), Observed electric fields associated with lightning initiation, *Geophys. Res. Lett.*, *32*, L03813, doi:10.1029/2004GL021802.
- Trahtenhertz, V. Y., and D. I. Iudin (2005), Actual problems of thundercloud electrodynamics [In Russian], *Izvestiya Vuzov Radiofizika Radiophys. Quantum Electron.*, *48*(9), 810–822.
- Zalesak, S. T. (1979), Fully multidimensional flux-corrected transport algorithms for fluids, *J. Comput. Phys.*, *31*, 335–365.
- Zheleznyak, M. B., A. K. Mnatsakanyan, and S. V. Sizykh (1982), Photoionization of the nitrogen and oxygen mixtures by the gas-discharge radiation, *High Temp.*, *20*, 357–362.